Analytical and numerical models for estimates of power loads from calorimetric measurements

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Abstract

The total power loads applied to inertially cooled components are commonly calculated starting from calorimetric measurements as cooling fluid temperature increase and flow rate during and after the power load application. In this paper, some generalizations to the standard models of measure interpretation are introduced.

A method is presented to take into account the effect of thermal conduction and radiation between the component and the surrounding material. It consists in evaluating the conduction and convection thermal resistances and the radiation heat flux by Computational Fluid Dynamics (CFD) numerical models. An analytical model permits to use these evaluations to obtain, from the measured convection flux, the corresponding fluxes by conduction and radiation.

Sometimes the durations of pulses and duty cycles are such that the calorimetric measurements after one pulse are actually influenced by the previously applied pulse. A correction factor is introduced to cancel this systematic error.

The cooling down phase can be long compared to the duty cycles between the pulses, and consequently the signals cannot be completely recorded till the thermal equilibrium. In order to evaluate the total heat flux on a component also in this case, a model is presented which considers a proper extrapolation curve to estimate the integral of the energy exhausted by the cooling systems.

As an example, the models introduced in the paper have been used to interpret experimental measurements carried out on the RADI experiment at IPP Garching and some results are presented in the paper.

Key words: analytical, numerical, models, calorimetric, measurements

1. Introduction

During experimental sessions in fusion experiments, some components can be subjected to high heat fluxes, that can lead to damages if the heat is not exhausted by a proper cooling system. Among these high heat flux components, there are for example some parts of the Neutral Beam Injectors (drivers, back plate, acceleration grids, neutralizer, residual ion dump, calorimeter), of the Radio Frequency antennas, of the Divertor and of the Blanket.

Measuring these heat fluxes can be useful for the design and correct dimensioning of other experiments, and to have information on the energy fluxes inside the machine [1,2]. In particular, measuring the energy absorbed by these components during short pulses can be useful before operating the experiment with long pulses, which are much more critical for the risk of damage. This paper deals with calorimetric measurements during pulsed sessions, which are common in fusion experiments. Also for the experiments that are designed for long pulse, and hence with a cooling system able to work in steady state conditions, during the first phase the pulses are short in order to test the machine; in this phase it is interesting to know the amount of power (and percentage on the total power) absorbed by the most critical components.

A simplified scheme of a generic actively cooled component subjected to heat flux is shown in Fig. 1a. A certain flow $\dot{m}$ of coolant (water most of times) exhausts the heat absorbed by the component. Some phases, shown in Fig. 1b, can be identified:

- the pulse phase ($t_1 \rightarrow t_2$), when the heat load is applied to the component. The outlet temperature T_{out} sensible increases. Also the inlet temperature T_{in} can generally increase, due to the limited cooling capabilities of the heat exchanger.
- a second phase ($t_2 \rightarrow t_3$) when T_{out} increases to the maximum value. This is generally reached after the end of the pulse.
- a third phase ($t_3 \rightarrow t_4$) when T_{out} decreases with a non exponential shape.
- a fourth phase ($t_4 \rightarrow t_5$) when T_{out} decreases with a

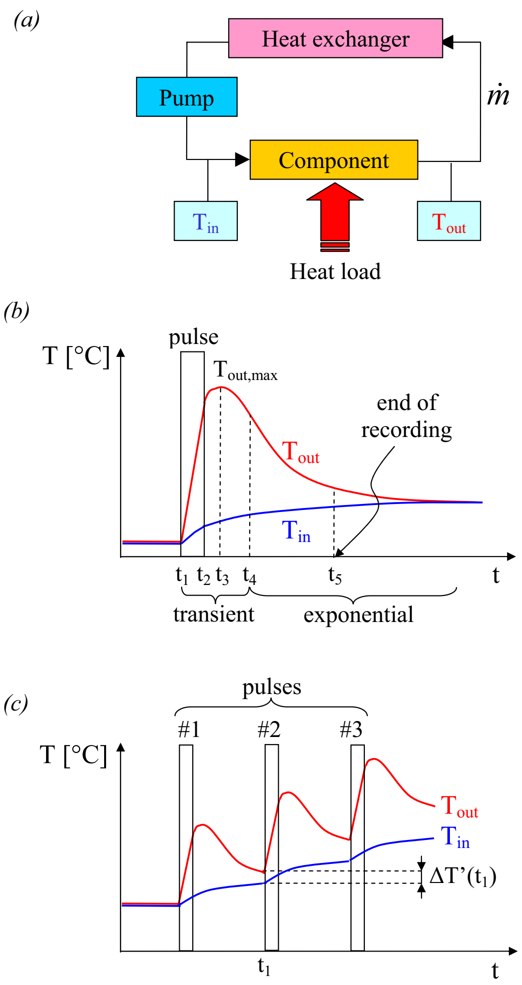

Fig. 1. (a) Cooling scheme of a generic component; (b) Scheme of a generic pulse; (c) Scheme of a generic experimental session

quasi-exponential shape.

The energy absorbed by the component during the pulse is usually calculated by a numerical integral of the heat exhausted by water between t_1 and t_5 (see for example [3]):

$$\begin{aligned} E &= \int_{t_1}^{t_5} q(t)dt = \int_{t_1}^{t_5} \dot{m} \cdot C_{fluid} \cdot (T_{out} - T_{in}) \cdot dt \\ &= \dot{m} \cdot C_{fluid} \int_{t_1}^{t_5} (T_{out} - T_{in}) \cdot dt \end{aligned} \quad (1)$$

where:
- $\dot{m}$ is coolant flow
- C_{fluid} is the heat capacity of the coolant
- $\int_{t_1}^{t_5} (T_{out} - T_{in})dt$ is represented by the shaded area in Fig. 1b

This approach is straightforward but affected by some approximations:
- The heat exhausted by conduction and radiation are neglected.
- As the cooling phase is not completely recorded (in principle, it is lasting an infinite time), there could be a not negligible area between the curves after the instant t_5
- The effect of the previous pulse ($\Delta T'$ in Fig. 1c) is not taken into account

This paper presents an analytical model (equivalent electric scheme) aiming at removing these approximations, and a numerical model (CFD) for a better estimation of the cooling parameters.

2. Estimation of thermal conductances

In order to consider the effects of thermal conduction and radiation, the equivalent electric scheme of Fig. 2a is introduced. This is a simplified analytical scheme that does not consider the thermal gradient inside the component. Both the conduction and convection processes are considered to occur between the average temperature of the component and the average temperature of water. This is an acceptable approximation if the component has a small temperature gradient (this happens in components made of good conductive materials, like copper) and if the water has a small temperature increase $T_{out} - T_{in}$ (this happens in most of high performance cooling systems).

The thermal energy is accumulated during the pulse in a thermal capacitance $m \cdot C_{comp}$, where m is the mass of the component and C_{comp} is the specific heat of the component material. The convection and conduction heat fluxes $q_\lambda(t)$ and $q_\alpha(t)$ are both proportional to the difference ΔT_{avg} between the average temperature of the component $T_{fluid,avg} + \Delta T_{avg}$ and the water average temperature $T_{fluid,avg}$. Analytically, the corresponding conductances G_λ and G_α can be estimated as:

$$G_\lambda = \frac{\lambda_{solid} \cdot A_\lambda}{s_\lambda} \quad (2)$$

$$G_\alpha = \alpha \cdot A_\alpha = 0.027 \cdot Re^{0.8} \cdot Pr^{\frac{1}{3}} \cdot \frac{\lambda_{fluid}}{D_h} \cdot A_\alpha \quad (3)$$

where:
- λ_{solid} is the thermal conductivity of the solid connection material
- A_λ and s_λ are the area and thickness of the connection
- α is the convective heat exchange factor, calculated in this case with the Sieder-Tate formula [4]
- A_α is the heat exchange area for convection (the interface area between material and coolant)
- $Re = \frac{v \cdot D_h}{\nu}$ is the Reynolds number
- $Pr = \frac{C_{fluid} \cdot \mu}{\lambda_{fluid}}$ is the Prandtl number
- v is fluid velocity inside the channel
- D_h is the hydraulic diameter of the channel
- ν is the cinematic fluid viscosity
- C_{fluid} is the fluid specific heat
- μ is the fluid dynamic viscosity
- λ_{fluid} is the thermal conductivity of the cooling fluid

A more precise way to estimate these two parameters is to perform a static CFD analysis, where the two conductances are calculated as ratio of the corresponding heat fluxes and the calculated ΔT_{avg}:

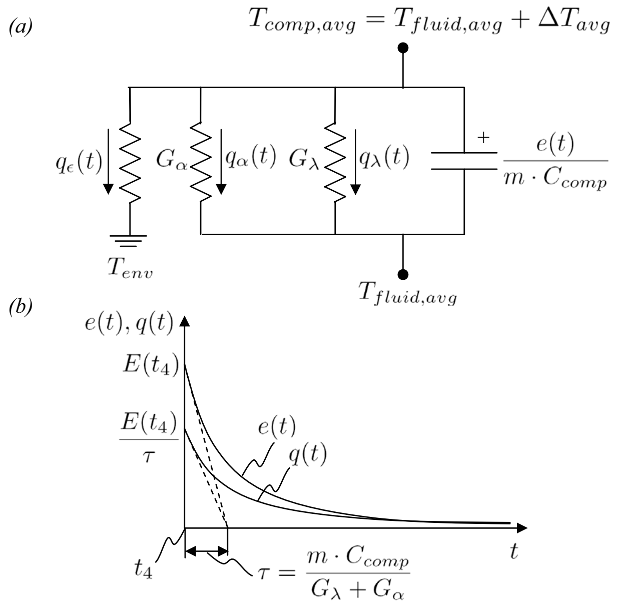

Fig. 2. Equivalent electric model of a component inertially cooled by conduction, convection and radiation: (a) Equivalent electric circuit; (b) Energy and heat curves (radiation neglected).

$$G_\lambda = \frac{q_\lambda}{\Delta T_{avg}} \tag{4}$$

$$G_\alpha = \frac{q_\alpha}{\Delta T_{avg}} \tag{5}$$

This numerical approach permits to take into account several aspects not considered with the analytical one, like:

- The temperature dependance of the fluid and solid properties
- The interaction between conduction and convection phenomena
- The thermal behaviour of the component. In particular, the surface temperature, that can be useful for the estimation of the heat transfer by radiation
- Possible thermal resistances due to a non perfect contact between materials

The radiation heat flux has a more complex behaviour, so it is not possible to identify a corresponding conductance, and the heat flux is estimated with the approach described in paragraph 4.

3. Estimation of the cooling time constant

Considering the scheme of Fig. 2a, it can be demonstrated that, if the cooling given by radiation is neglected ($q_\epsilon \ll q_\lambda + q_\alpha$), the component temperature has an exponential decay. In fact the ΔT_{avg} between average temperature of the cooling fluid $T_{fluid,avg}$ and component $T_{comp,avg}$ can be written as $\frac{e(t)}{m \cdot C_{comp}}$ but also as $\frac{q(t)}{G_\lambda + G_\alpha}$. Solving the related differential equation

$$\frac{1}{m \cdot C_{comp}} \cdot e(t) - \frac{1}{G_\lambda + G_\alpha} \cdot \frac{de(t)}{dt} = 0 \tag{6}$$

it can be found that the energy stored in the component $e(t)$ and the heat flux $q(t)$ have both an exponential decay:

$$e(t) = E(t_4) \cdot e^{-\frac{t-t_4}{\tau}} \tag{7}$$

$$q(t) = E(t_4) \cdot \frac{1}{\tau} \cdot e^{-\frac{t-t_4}{\tau}} \tag{8}$$

where $E(t_4)$ is the thermal energy stored in the component at the beginning of the exponential phase, and the time constant τ is equal to $\frac{m \cdot C_{comp}}{G_\lambda + G_\alpha}$. The average temperature $T_{comp,avg}(t)$, being quasi proportional to the total energy stored in the component $e(t)$, has also a quasi exponential decay with the same time constant τ. The outlet temperature can be written as

$$T_{out}(t) = T_{in}(t) + \frac{q(t)}{\dot{m} \cdot C_{fluid}} \tag{9}$$

and hence the function

$$\Delta T_{water}(t) = T_{out}(t) - T_{in}(t) = \frac{E(t_4)}{\tau \cdot \dot{m} \cdot C_{fluid}} \cdot e^{-\frac{t-t_4}{\tau}} \tag{10}$$

has, with some approximation due the hypotheses above, the same decay time constant of $e(t)$, $q(t)$ and $T_{comp,avg}(t)$. Hence, the time constant τ of the cooling process can be evaluated by interpolation of the function $T_{out}(t) - T_{in}(t)$ considering the experimental values between t_4 and t_5.

4. Estimation of the heat fluxes

In every moment, the heat exhausted by convection cooling (normally the main cooling system of the component) can be calculated as:

$$q_\alpha(t) = \dot{m} \cdot C_{fluid} \cdot (T_{out}(t) - T_{in}(t)) \tag{11}$$

A formula to estimate the heat exhausted by water related to the previous pulse is here proposed:

$$q'_\alpha(t) = \dot{m} \cdot C_{fluid} \cdot \Delta T'(t) \tag{12}$$

where the following hypotheses can be made:

- the function $\Delta T'(t)$ have an exponential decay with the same time constant τ of the function $T_{out}(t) - T_{in}(t)$
- $\Delta T'(t_1) = T_{out}(t_1) - T_{in}(t_1)$

Taking into account a non complete recording of the cooling down phase and the effect of the previous pulse, the energy exhausted by convection can be then estimated as

$$E_\alpha = \int_{t_1}^{t_4} q_\alpha(t) \cdot dt + \int_{t_4}^{+\infty} q_\alpha(t) \cdot dt + \int_{t_1}^{+\infty} q'_\alpha(t) \cdot dt \tag{13}$$

The first integral (energy exhausted during the transient phase $t_1 \rightarrow t_4$) can be numerically evaluated from the signals, while the second (energy exhausted during the exponential cooling down phase) and third (correction factor to take into account the effect of the previous pulse) can be analytically calculated using the cooling time constant (evaluated with the method described in paragraph 3):

$$\begin{aligned} E_\alpha = & \int_{t_1}^{t_4} \dot{m} \cdot C_{fluid} \cdot (T_{out}(t) - T_{in}(t)) \cdot dt \\ & + \dot{m} \cdot C_{fluid} \cdot (T_{out}(t_4) - T_{in}(t_4)) \cdot \tau \\ & - \dot{m} \cdot C_{fluid} \cdot (T_{out}(t_1) - T_{in}(t_1)) \cdot \tau \end{aligned} \quad (14)$$

Following the scheme of Fig. 2a, the energy exhausted by conduction is proportional at every moment to the heat exhausted by convection and hence can be estimated as:

$$E_\lambda = E_\alpha \cdot \frac{G_\lambda}{G_\alpha} \quad (15)$$

As regards to the heat exchanged by thermal radiation, it can be estimated with the Stefan-Boltzmann formula:

$$q_\epsilon(t) = \epsilon(T) \cdot \sigma \cdot A_\epsilon \cdot \left[(T_{env} + \Delta T_{surf}(t))^4 - T_{env}^4\right] \quad (16)$$

where:

- $\epsilon(T)$ is the emissivity (ratio of energy radiated by a particular material to energy radiated by a black body at the same temperature)
- σ is the Stefan-Boltzmann constant
- A_ϵ is the radiation area
- T_{env} is the environment absolute temperature
- $T_{env} + \Delta T_{surf}(t)$ is the absolute average surface temperature of the component

The energy exhausted by radiation can be calculated by integration of this heat.

$$E_\epsilon = \int_{t_1}^{t_3} q_\epsilon(t) \cdot dt + \int_{t_3}^{+\infty} q_\epsilon(t) \cdot dt \quad (17)$$

The average surface temperature $T_{env} + \Delta T_{surf}(t)$ is reached at the instant t_3 (see Fig. 1b) and can be calculated with a proper CFD model, as exemplified in paragraph 5. Assuming the hypotheses of a linear increase of the surface temperature in the phase ($t_1 \rightarrow t_3$) and of an exponential decay after t_3, this integral is solved as:

$$\begin{aligned} E_\epsilon = & \epsilon(T) \cdot \sigma \cdot A_\epsilon \cdot [2 \cdot T_{env}^3 \cdot \Delta T_{surf}(t_3) \cdot (t_3 - t_1 + 2 \cdot \tau) \\ & + 2 \cdot T_{env}^2 \cdot \Delta T_{surf}^2(t_3) \cdot (t_3 - t_1 + \frac{3}{2} \cdot \tau) \\ & + T_{env} \cdot \Delta T_{surf}^3(t_3) \cdot (t_3 - t_1 + \frac{4}{3} \cdot \tau) \\ & + \frac{1}{5} \cdot \Delta T_{surf}^4(t_3) \cdot (t_3 - t_1 + \frac{5}{4} \cdot \tau)] \end{aligned} \quad (18)$$

The total energy absorbed by the component during the pulse can be then calculated as:

$$E_{TOT} = E_\lambda + E_\alpha + E_\epsilon \quad (19)$$

5. Application to the RADI Faraday shield

RADI is one of the experimental Radio Frequency (RF) ion sources at IPP Garching. It has approximately the same width of the ion source foreseen for ITER NBI and half the height, and is particularly dedicated to the testing of plasma uniformity in large size RF ion sources [5]. It features four RF drivers, with the function of producing a hydrogen or deuterium by means of RF fields. Part of the power input by the RF coils is absorbed by the Faraday shield, a copper component that protects the ceramic insulator and is heated by eddy currents and by thermal radiation from the plasma inside the source. For the understanding of the physics of this experiment and for the design of other ion sources (like the one for ITER), it is interesting to know the amount of power deposed on this component [6,7]. In order to make this with a good approximation, the models described in this paper were applied.

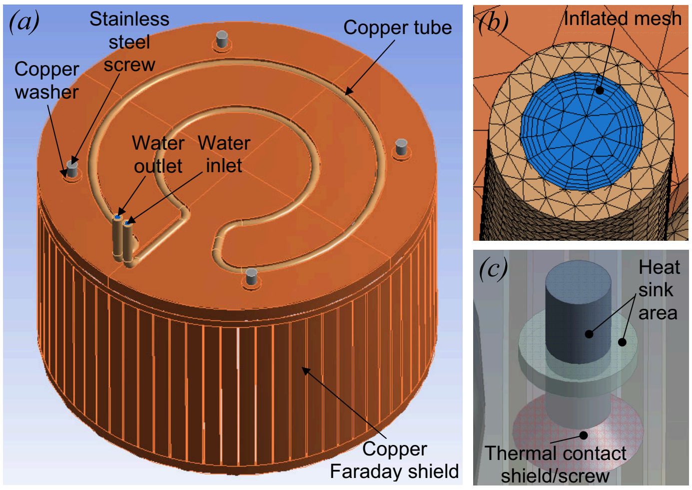

Fig. 3. CFX model of the RADI Faraday shield: (a) Overall Model; (b) Mesh view with focus on the inflated mesh on the boundary layer: this mesh permits to satisfy the quality requirements for k-ϵ turbulence model; (c) Screw detail: a perfect thermal contact is applied between screw and copper on the conical cap, while a heat sink condition is applied to the external face of the washer and to the threaded part of the screw (the back cover is considered to have a temperature equal to T_{in} and an infinite thermal capacitance)

A Computational Fluid Dynamics (CFD) model of the Faraday shield was created with the software CFX (see Fig. 3). This comprises the copper shield, the cooling tube, the water inside the tube and the screws that connect the component to the aluminium back cover. The mesh is inflated on the water side of the water/tube interface, in order to better simulate the heat exchange between coolant and solid. The component is also cooled by conduction, as it is in contact with the back cover by means of four copper washers and four stainless steel screws.

Fig. 4 shows the outlet water temperature measured during and after a RADI pulse. The inlet and outlet temperature data recorded between t_4 and t_5 were used to estimate the cooling time constant τ like explained in paragraph 3. This was used to calculate the energy exhausted by convection with formula 14.

A steady state thermal-fluid-dynamics analysis was then performed the CFD model of the Faraday shield, at the instant t_3, that corresponds to the highest temperature. The

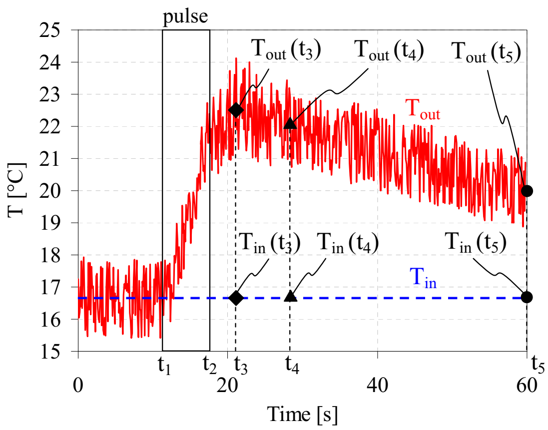

Fig. 4. Outlet temperature measurements on a RADI Faraday shield cooling water during and after pulse 13704. A noise corresponding to about 2°C is present on the signal. The inlet temperature, not measurable for this component, is here supposed equal to the average outlet temperature before the pulse.

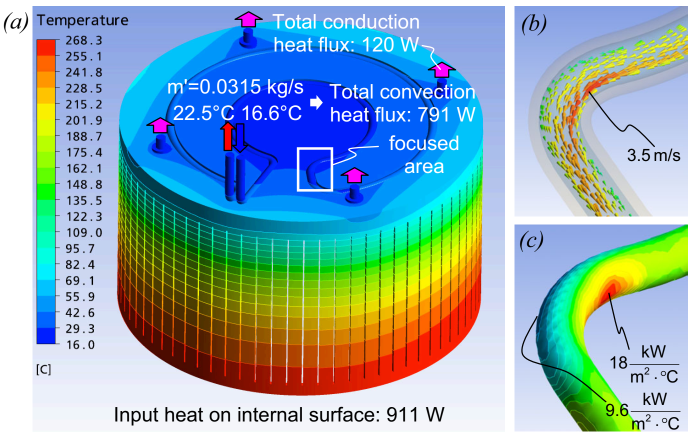

Fig. 5. Main results of the thermo-fluid-dynamics analysis of the RADI Faraday shield: (a) Temperature plot at the moment of maximum temperature (instant t_3); (b) Vector plot of the fluid velocity in the focused area; (c) Corresponding convective heat transfer coefficient at the water/tube interface

inlet water temperature was then set at 16.6°C, while the input heat load was applied on the internal surface of the Faraday shield (area where it is foreseen to be applied also on the reality), with a total amount set in order to obtain an outlet temperature of 22.5°C. In this way, the conductive and convective heat fluxes q_λ and q_α could be estimated (see Fig. 5). The two corresponding conductances G_λ and G_α were then estimated with formulae 4 and 5. Finally, the energy exhausted by conduction and radiation were calculated with formulae 15 and 17. For the latter, the absolute environment temperature T_{env} was considered equal to the water inlet temperature 289.7 K.

The main results of this postprocessing based on both analytical and numerical models are reported in Tab. 1, where they are compared with the ones calculated using a purely analytical postprocessing.

Table 1
Summary of the heat load estimations obtained for pulse 13704 in RADI by using a purely analytical postprocessing and using both the analytical and numerical models.

	Analytical	Analytical + Numerical (CFD)
$G_\lambda[W/°C]$	16	20
$G_\alpha[W/°C]$	129	132
$\tau[s]$	87	87
$E_\lambda[kJ]$	9.3	11.4
$E_\alpha[kJ]$	75	75
$E_\epsilon[kJ]$	/	1.3

6. Conclusions

This paper presents a novel approach to estimate the heat absorbed by an inertially cooled component during pulsed sessions. This is made by considering analytical and numerical models, which permit:
– to calculate the heat fluxes by conduction and radiation, from the experimental data on convection
– to take into account the whole cooling phase, also in the case that it is not completely recorded by the diagnostics
– to consider the effect of a previous pulse

The models have been applied to the calorimetric measurements on the RADI ion source at IPP, with good results. The total energy estimated is in agreement with [6].

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